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Solar Sailing—A Practical Method of Propulsion Within the Solar System

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It is shown that commercially available metallized plastic film can be used as a solar radiation pressure sail for propulsion of space vehicles within the solar system. The method of propulsion is of negligible cost and is perhaps more powerful than many competing schemes.

IT IS difficult to exaggerate the importance of solar radiation pressure for the propulsion of satellites or space ships within the solar system; but since I have never seen any allusion to this powerful method, while less practical and more difficult schemes are frequently cited, I feel it desirable to publish this paper.

The principle involved is simply to make use of the pressure of the sun's light on a sail to propel a space ship as desired through the solar system. What is important is the area of sail per unit mass, and if, just as a practical example, we use a commercially available 0.1-mil-thick plastic sail equal in mass to the rest of the "space ship," the mass per unit area will be 5×10^{-4} g/cm^2. The sail may be aluminized without significant additional mass, in which case the sun's radiation may be reflected, thereby doubling the maximum pressure which may be exerted on the sail. The magnitude of this pressure on a normally oriented sail is $P = 2W/c$, where W is the energy incident per cm^2 per sec, and c is the velocity of light. At the earth's orbit $W = 2$ cal/cm^2/min or 1.3×10^6 erg/sec/cm^2, so that $P = 0.8 \times 10^{-4}$ dyne/cm^2. The acceleration of the space ship is, therefore, $a = p/\sigma$, where σ is the mass/cm^2 and in the example is 5×10^{-4} g/cm^2. Therefore $a = 0.16$ cm/sec^2. Thus on a space ship so constituted one may obtain an acceleration of as much as 1.6×10^{-4} times that of gravity even with a gross 0.1-mil sail.

The thrust may be varied from the maximum calculated, smoothly almost to zero by manipulation of the "shroud lines"; and the thrust may be directed almost without loss in magnitude to any angle within 45 deg of the radius vector from the sun, while smaller thrust (as $\cos \theta$) is available up to 90 deg from the radius vector.

Although the acceleration is numerically small, the velocity changes in reasonable times by significant amounts. Thus $\Delta v = 1.4 \times 10^4$ cm/sec/day, so that the escape velocity from the earth (8×10^5 cm/sec) may be acquired in two months, and a velocity large enough to escape from the solar system entirely may be reached in a time of the order of a year.

Before comparing the merits of solar sailing with those of ion propulsion, hydrogen expulsion by solar batteries, or nuclear or thermonuclear reactors, let us discuss two situations in which solar sailing might well be used. To have a definite example, consider an earth satellite with reflective sails, each weighing 20 lb (10 kg). The sail will have an area

$$\frac{2 \times 10^4 \text{ g}}{5 \times 10^{-4} \text{ g/cm}^2} = 4 \times 10^7 \text{ cm}^2$$

and will be more or less a circle of plastic film 70 meters in diameter. There are, of course, no requirements as to absence of holes, tears, flaws, etc. The reflective coat plays a perhaps important role in eliminating electrostatic forces which might easily prevent opening of the sail. The sail may be attached to the ship by "shroud ribbons" ∼200 meters long. Since the forces on the shroud ribbon will not exceed 2 grams weight, the ribbons may be narrow strips of the same material as the sail. Let us for the moment grant that the sail may be expelled from its packing container after the "space ship" is in a satellite orbit at reasonable altitude. It is of interest to compute the time required for the sail to "fill with sunlight." This is the time for the sail to go from a flat circle to a dished configuration of the order of 10 meters deep and is

$$t_f = \left(\frac{2 \times 10^3}{0.32}\right)^{1/2} = 80 \text{ sec}$$

Thus the sail may be furled and unfurled quickly compared with the 90-min period of its orbit around the earth.

The simplest program for increasing the altitude of the satellite, at the same time increasing its energy and *reducing* its velocity about the earth, is to furl the sail by slacking half the shroud lines while the satellite approaches the sun and to unfurl the sail while receding from the sun. Clearly then, the mean projected acceleration along the velocity vector is $\pi^{-1} \times 0.16$ cm/sec^2 or 5×10^{-2} cm/sec^2, and the rate of decrease of orbital velocity is then $\dot{v} = 5 \times 10^{-2}$ cm/sec^2. Thus, an initial velocity of 8×10^5 cm/sec will decrease to zero in a time on the order of $(8 \times 10^5)/(5 \times 10^{-2}) = 1.6 \times 10^7$ sec, or 6 months. The orbit eccentricity grows during this time, and, depending on details, the altitude at perigee may remain constant or increase.

It should be noted parenthetically that the torque exerted on a satellite by radiation pressure is ordinarily larger than that due to differential gravity effects. Indeed, if half of a 20-in.-diam Vanguard sphere is painted black, while the other half is reflecting, and if the moment of inertia is 2×10^6 gm cm^2, the torque is of magnitude ∼1 dyne-cm, and angular acceleration will be $\sim 10^{-6}$ radians per sec. The sphere will, therefore, revolve one radian in ∼30 min due to this cause. For this same sphere with maximum gravitational quadrupole (two 5-kg masses at opposite ends of a diameter), the differential gravity torque is 10 dyne-cm at 45 deg inclination. On this small-area device, the torque due to radiation pressure is not negligible. For less symmetrical devices (cones, etc.) a stable equilibrium may be reached pointing into the sun, while damping of long-period oscillations about this equilibrium may be achieved by slightly viscous fluids in some portion of the satellite.

The energy required for operating the shroud lines is at most (2×10^3 dynes) $\times$ (10^4 cm) or ∼2 joules each 90 min, even if a nonconservative actuator is used. This is 20 milliwatts, while the kinetic energy delivered to the satellite by the sail averages 60 watts. The maximum power falling on the sail is $\sim 4 \times 10^6$ watts (4 megawatts) so that it is evident that the propulsion efficiency is not very high ($\sim 10^{-5}$ or, in fact, v/c). A solar battery ∼1cm^2 area supplies enough power to manipulate the shroud lines.

As a second example let us consider a space ship of the same fairly high mass per unit area (0.1-mil sail of mass equal to that of the rest of the ship) $\sim 5 \times 10^{-4}$ g/cm^2. Now we want to go from the earth's orbit to that of Venus and back again.

The optimum sail tilt to go slowly with a small sail from one circular orbit to another is $\theta = \sin^{-1} \sqrt{3}/3$, since this angle gives the maximum component along the velocity vector and thus allows the ship to lose or gain energy as rapidly as

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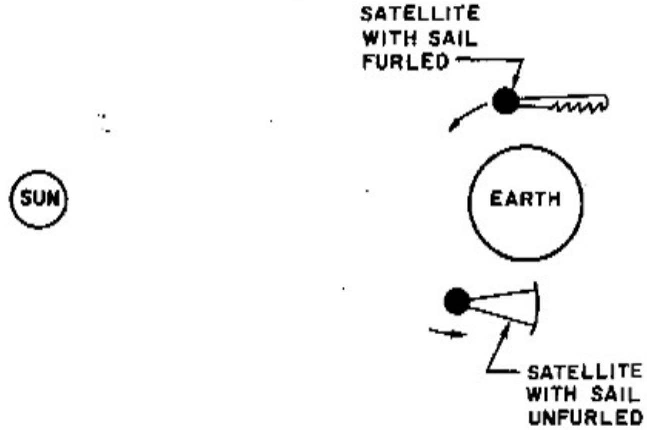

Fig. 1 Use of the solar sail to increase the altitude of a satellite and to escape the earth

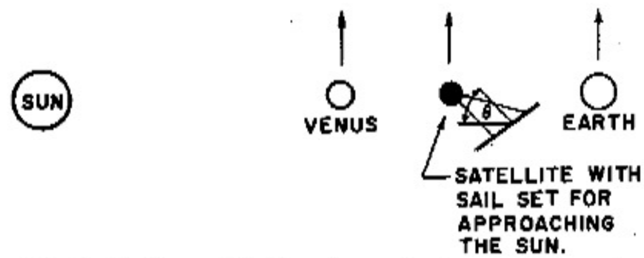

Fig. 2 Cruising within the solar system by use of the sail

possible. In order to move in to Venus we must *brake* the ship to *increase* the orbital velocity (because the potential energy is twice the kinetic energy and negative). In the field of our sun with the space ship given above we have

$$\dot{v} = \frac{(0.8) \times (0.8 \times 10^{-4})}{(5 \times 10^{-4})} \left(\frac{r_0}{r}\right)^2 = 0.1 \left(\frac{r_0}{r}\right)^2 \text{ cm sec}^{-2}$$

where r/r_0 is the distance of the ship from the sun measured in earth's orbit radii ($\sim 1.5 \times 10^{13}$ cm). But

$$v = 2.4 \times 10^6 \left(\frac{r_0}{r}\right)^{1/2}$$

Thus we find for the rate of change of r with time

$$\frac{\dot{r}}{r} = 10^{-7} \left(\frac{r_0}{r}\right)^{3/2}$$

showing that the orbit is an equiangular spiral, opening or closing by a large factor each turn (indeed, by a factor ~ 20 per year if the initial conditions were right for the motion to be an equiangular spiral in this noncentral-force field).

The orbit equations are not difficult to solve in detail, but we may here be content with observing that the radiation pressure in our example is one-half the gravitational force between sun and space ship, so that the space ship is entirely free to go "tacking" about in the solar system, arriving at any specified radius in times considerably less than a "year" for the planet at that radius. For instance, one can go to Venus and back in less than an ordinary year with this first gross sail.

Compared with escape from the earth, the efficiency of the sail rises for operation near the earth's orbit, from $\sim 10^{-5}$ for earth satellite propulsion to $\sim 10^{-4}$ for solar navigating, since the pertinent velocity is higher and the duty cycle more favorable. We may compare alternative schemes for propulsion, the one currently most popular being, perhaps, acceleration of the interplanetary ions ($\sim 10^3/\text{cm}^3$) by an accelerator in the vehicle, powered by solar energy or nuclear or thermonuclear reactor. To achieve the 4×10^3 dynes thrust contemplated here with gentle use of a sail, one would have to accelerate to, say, 100 kev energy one ampere of interplanetary protons. This would require only 100 kilowatts of power rather than the 4 megawatts of sunshine reflected (lower, of course, by the ratio of proton exhaust velocity to light velocity); but the space ship swallows up only $(2.4 \times 10^6) \times (10^3) \times (A)$ protons per sec, where A is the frontal area of the accelerator. Thus $A \geq 10^9$ cm^2, and the space ship must have a maw larger than the sail contemplated here. The use of a thermonuclear reactor to eject ~ 1 gram of hydrogen per day at 100 kev (in our 20-kg ship) would be an eminently satisfactory solution if such reactors existed and were of zero mass. Even so, one must eject one's own mass every 30 years to compete with the solar sail for cruising.

It is obvious that there are considerable difficulties connected with space travel, but those connected with the sail appear relatively small. We note that a reasonable chemical fuel with exhaust velocity ~ 2 km/sec, requiring a ratio of take-off mass to final mass of $e^{v/V}$, were V is the jet velocity and v the desired final rocket velocity, or a mass ratio of $e^4 = 250$ to reach 11 km/sec (escape velocity from the earth) would require to reach 24 km/sec a mass ratio of 1.6×10^5, if rocket engines and tanks had zero mass. In actuality the mass ratios are very much larger.

Possible Improvements

It should be no great trick to make large areas of sail of thickness 2×10^{-5} cm (as is used for beta-ray source backing), thus reaching a space ship area per gram twelve times smaller than that considered above. With this improved sail a satellite could escape the earth within a week, and could fall to Venus in less than a month and come back in a week (at high velocity, albeit).