one of the roots to the characteristic equation. This is also a sufficient condition for amplification to occur.

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Superconducting Lines for the Transmission of Large Amounts of Electrical Power over Great Distances

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***Abstract*—As an application of high-field, high-current superconductors we sketch the design of a power transmission line to carry 100 GW (10^{11} watts) of direct current over a distance of 1000 km. (It is interesting to note that the present peak power generating capacity of the United States is approximately 200 GW, or just twice the capacity of the proposed line.) Such a line, in contrast to one made of ordinary metal, would dissipate none of the power transmitted through it, although it is necessary to tap power from the line for refrigeration. The consequences of negligible transmission loss are substantial: power transmission would be more economical than the present practice of shipping coal to the region in which electricity is generated and consumed; generating-plant site selection could be made almost entirely on economic considerations; at the same time, thermal and air-pollution problems could be minimized; novel power sources could be considered.**

The power line would be made of Nb_3Sn and would be refrigerated to 4°K. The power must be transmitted as direct current, rather than as alternating current, because the very large (comparatively) alternating-current losses would require excessive refrigeration capacity.

Specifically, we shall discuss a line at 200 kV carrying 0.5×10^6 A. The investment in the line will be approximately $806 million, or $8.06/kW. Of this, some $6.06/kW is line cost, the remainder being converter cost, which, of course, is the same for an ordinary dc line. In comparison with the shipping of coal, the investment cost would be repaid in ten months.

We have investigated in some detail the problems of refrigeration along the line, including those of heat leak through the wires which deliver power to customers at room temperature. The efficiency of the line is greater than 99.9 percent (power transmitted less the power drawn off to run refrigeration equipment, all divided by transmitted power).

While the technical discussion is probably correct, the cost figures do not include engineering expeditures and do not consider in detail the costs involved in providing the redundancy and safety factors for, say, a failure rate of one per ten years with a time of a few seconds to restore power.

This is not an engineering study but rather a preliminary exploration of feasibility. Provided satisfactory superconducting cable of the nature described can be developed, the use of superconducting lines for power transmission appears feasible. Whether it is necessary or desirable is another matter entirely.

INTRODUCTION

Before World War II, there was little incentive for the electric utility companies to develop means of long-distance transmission of electrical energy. In fact, the only high-voltage long lines in existence were those required to transport power from hydroelectric sources to population centers. Electrical energy transmission over long distances was avoided mostly because of the high cost of transmission over lines (a compromise between power loss in a small conductor and the capital expense of a large-diameter line). It was more economical to build generating stations near the major consumption centers, and to transport the energy to the generating stations in some other way; for example, as coal by barge or railway. This procedure was also consistent with the structure of the electric utility industry, which not long ago consisted of many comparatively small local companies, each serving its own area, quite independently of one another.

The recent (and future projected) growth of the utility

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industry has forced a re-examination of the economics of past practice. In particular, although the costs of coal transportation by railway have been decreasing, this cost is still substantial.[1] Furthermore, the utilities have recently become aware of the advantages of power pooling. By tying together formerly independent power systems they can save in reserve capacity (particularly if the systems are in different regions of the country), because peak loads, for example, occur at different times of day, or in different seasons. To take advantage of these possible economies. facilities must exist for the transmission of very large blocks of electrical energy over long distances at reasonable cost.

Other problems which face the utility industry also require for their solution economical means of electrical energy transmission. Among these are generating-station site location, and full utilization of existing or of novel power sources. The location of fossil-fuel plants near or in high-population-density areas has disadvantages. Suitable sites may be unavailable or very expensive; air pollution or thermal pollution problems may limit the generating capacity below optimum size; there is still much resistance to placing nuclear power plants in congested areas, because of possible dangers, no matter how improbable they may be. Distant sites make available economies of scale which are particularly important in nuclear plants.

Should it be possible to transmit large amounts of electrical energy with negligible loss, fossil-fuel as well as nuclear plants could be placed so as to offer no hazard to urban areas, and the choice of location could be made entirely on economic considerations; in particular, mine-mouth operation of steam plants would lead to large savings in coal transportation costs.

Economical transmission would also make possible serious consideration of alternative sources of energy; for example, one might build large solar generators in deserts or tap sources of hydroelectric power much more distant than those now considered practical for development.

In this paper we consider the problem of economical electrical energy transmission; in particular, the problem of transmitting very large blocks (100 GW) over long distances (1000 km). These numbers, while large, are not unreasonably so, since inter-ties of 4-GW capacity over such a distance are currently under serious consideration [3]. Large block transmission is especially important since, whereas satisfactory conventional solutions exist for the transmission of multi-megawatt blocks over distances of ~500 km, this is not the case, as will be made clear in this paper, for 100 GW over 1000 km.

There are, as in every engineering problem, a number of possible alternative solutions, the choice among which must be made on the basis of cost or return. The first alternative, of course, is not to transmit electrical energy at all but rather to ship coal. The shipment of coal does offer some advantages; for example, easy storage of energy near the consumer, which eliminates, or at least minimizes, the peak-load problem on the transmission line, and provides security against short-term (few-week) interruptions of the transportation system.

The transportation cost of coal for a power transmission task of this magnitude, however, exceeds $1 billion per year. This cost is large; therefore, considerable effort is justified to reduce it.

Another alternative is conventional (probably dc) long-distance EHV transmission by ordinary metallic conductors. This solution has the advantage of the use of existing and proven technology. On the other hand this, too, is expensive. Lower bounds for cost involving such a line are $670 million for the conductor, perhaps several times this amount for towers and, most important, a line loss of ~$340 million per year. To these we must add converter costs, if the transmission is to be at dc (Appendix A).

We propose as a solution a superconducting line; i.e., a transmission system using superconductors to carry current. The superconductors would, of course, be refrigerated so that their resistivity is zero. The major power loss then would be the power required to maintain the line at 4°K.

In what follows we shall sketch the design of such a line.

DETAILS OF THE PROPOSED TRANSMISSION LINE

The proposed transmission line consists of two insulated superconducting cables which are maintained at 4°K (the boiling point of liquid helium at one atmosphere), a convenient working temperature.

The power transported down the line is still, of course, proportional to the voltage difference between the two cables, and to the current flowing (with essentially zero dissipation) in the cables.

The design problem can be divided into three rather natural, but obviously very much interdependent parts: the superconducting cable itself, the refrigeration system, and the transmission line as a whole, including its relationship to existing facilities.

The cable problem involves: the choice of an appropriate superconductor and of cable dimensions based on the necessary currents and voltages; demonstration of the need for dc transmission, which is required by the economics of the refrigeration system and has far-reaching consequences for the line as a whole; finally, some details of construction.

The refrigeration system uses the principles of the common Dewar flask (Thermos bottle); i.e., the primary coolant is liquid helium (He), the insulation is vacuum, and the radiant heat shield is cooled by liquid nitrogen (N_2). The power to run the refrigerators is tapped from the line.

Finally, there are the problems of providing for redundancy and repair of the transmission line and the tie-in problems to ac systems at ordinary temperature.

A. Superconducting Cable

As the superconducting material from which the cable is made, we choose niobium-tin (Nb_3Sn). At temperatures

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[1] The average mine cost of coal is $4.50 per ton (see Howard [1]), whereas the average transportation cost (1962) was $3.37 per ton [1]. Since approximately 1.2 kWh of electrical energy is obtained per pound of coal, the transportation cost of coal is approximately 1.4 mills/kWh. This transportation cost is an appreciable fraction of busbar energy costs (1964) of 6.8 mills/kWh (Olmsted [2]).

low compared with its "critical temperature" ($T_c = 18°\mathrm{K}$), this material remains superconducting in a field of 100 000 G[2] while carrying current densities of 200 000 A/cm^2 [4]. (For a brief summary of properties of superconductors, see Appendix B.) If we choose the transmission voltage as 200 kV, the total current must be 0.5×10^6 A. Limiting the current density to 10^5 A/cm^2 implies a superconductor cross section of 5 cm^2. This choice insures that the self-magnetic field is below the critical one of 10^5 G, even for cable made entirely of superconductor; if the cable is "stabilized" [6] by the inclusion of low-resistivity aluminum or copper, which occupies space and thus increases the radius, the magnetic field will, then, be still smaller.

The choice of transmission voltage of 200 kV is obviously conservative since 345-kV conventional insulated underground lines are in use [7]. It is probable, since insulation problems are much eased at low temperatures, that the transmission voltage could be raised to 500 kV, thereby achieving a 2.5-fold increase in the power transmission capability of the line, or alternatively a 2.5-fold reduction in the current and thus in the cross section (and cost) of the superconductor.

Nearly all transmission lines in the world today transmit alternating current. The major reason is the ease with which the voltage can be changed ("transformed") from link to link along the system. Even so, there are reasons to prefer dc transmission under some circumstances. Insulation requirements are less severe with dc transmission and only two conductors are required per dc circuit rather than the three required in the ac case. Furthermore, the maximum practical length of ac cable is limited by the capacitive "charging" current, which, of course, does not exist on dc lines.

There are, however, far more compelling reasons for preferring dc transmission on *superconducting* lines.

Alternating currents exert alternating forces on flux lines in the superconductor causing irreversible motion of the flux lines (thus, loss). This produces heat. In fact, as we shall see below, normal heat leak into the line is $\sim 4 \times 10^{-4}$ W/cm. Hysteresis losses, if transmission were at 60 Hz, would exceed this by a factor or 10^8, making line refrigeration impossible. [For details, see Appendix C, (1) through (3).]

Note added in proof: A recent paper [21] demonstrates the unpracticability of ac transmission on refrigerated lines. Note, however, that the coolant circulation in Fig. 3 of that paper ("twin-duct jacket") is unsatisfactory because such a counter-current flow does not remove appreciable heat.

Even with a (nominally) dc line, fluctuating demand and particularly the initial loading of the line cause dissipation, largely as a result of hysteresis in flux motion. This loss is a serious design consideration if the line is not to serve only as a base supply, at constant load.

Fluctuating demand can readily be handled. Equation (4) (in Appendix C) gives the dissipation due to fluctuations as

$$H = 2 \times 10^{-15} (\Delta I)^3 F \quad \mathrm{W/cm} \tag{4}$$

where F and ΔI are the frequency and amplitude of the fluctuation, respectively. Thus, the switching on and off of individual customers of 1 MW or 5 A every millisecond produces a dissipation of only 2.5×10^{-10} W/cm.[3]

Turn-on is a more difficult problem. Equation (2) (in Appendix C) gives a total dissipation of 0.07 W/cm for current growing from zero to 0.5×10^6 A in one hour, exceeding the radiative heat leak by a factor of 200!

Electrical loading of the line begins after the line has been cooled to 4°K from room temperature, a process which requires approximately five days. In terms of such a time scale a turn-on time of one or even two days is not unreasonable. The dissipation for a one-day turn-on time will exceed normal heat leak by a factor of ten.

In any case, turn-on dissipation may be reduced by appropriate cable design. Thus, dissipation is reduced by a factor of g if the superconducting cable is indeed made of fine wires each of diameter g times smaller than the cable diameter. [A g of 200 equals the heat leak for a one-hour turn-on interval, and a g of 10, the heat leak for a one-day turn-on interval. A 3-cm cable diameter means an elementary diameter of 1.5×10^{-2} cm (or 3×10^{-1} cm for $g = 10$).]

The wires in a cable may be insulated from one another and effectively transposed over a 1-to-10-km length so that the wires initially at the core of the cable are on the outside for a similar distance, or the wires may be of shorter length in a normal-conducting (copper) matrix, the length of the wires being a compromise between steady-state loss and turn-on loss.

Clearly, there is room for considerable engineering in cable design. Since no lines of such capacity have yet been fabricated, the exact nature of the line depends upon the actual engineering compromises found to be necessary. The principle of the multistrand cable, however, is sound, as shown by the successful use of such cable in superconducting magnets [8].

Anelastic losses in ac transmission would also be large. (They would exceed normal heat leak by a factor of 10^4.) Because of the alternating current and, therefore, magnetic field, the pressure exerted on the conductor by the magnetic field varies between zero and its maximum value 120 times per second. Since the material is not perfectly elastic, some of this energy is lost as heat each cycle. An anelasticity of less than 10^{-5} would be required to have an elastic loss comparable with the heat leak. Such low anelasticity would be difficult to obtain (Appendix C). Furthermore, fatigue would also preclude the use of alternating current.

Another reason for dc transmission becomes clear when we realize that the characteristic impedance of the line is ~ 100 ohms, whereas the current voltage limitations require a load of about 0.4 Ω. The 100-Ω characteristic impedance implies that to carry 0.5×10^6 amperes on a long line requires a potential difference of 0.5×10^8 volts between cables and not 2×10^5 volts as in the dc case.

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[2] Nb_3Sn has been shown to remain superconducting in fields of 220 000 G [5].

[3] Note that if the cable is coated with copper, losses due to fluctuating demand, etc., will be eddy-current losses in the copper. The corresponding loss is 2×10^{-6} W/cm.

Unfortunately, superconducting cable is not produced or shipped in infinite lengths. This means that it is necessary to consider how to make joints. It will be difficult to make superconducting joints by the common methods since ordinary soft superconductors would be quenched anyhow in fields of 10^5 gauss. Therefore, we estimate the dissipation caused by pure normal-metal solder joints.

A cable cross section of 5 cm^2 means a mass of ~ 100 g/cm or 10 tons/km. Since 10 tons is a reasonable shipping weight, we must consider joints every km. A metal-film 10^{-3}-cm thick ($\rho = 10^{-8}$ $\Omega \cdot \text{cm}$ at 4°K) between butted superconducting sections 5 cm^2 in area will present a resistance of 10^{-12} ohms and will thus cause one-watt dissipation per joint. This might be handled without excessive temperature rise (~ 0.5°K), but the joint can in addition be skived (the ends cut at 6° angle to the axis of the bundle). The surface of the film is thus increased by a factor of ten, which results in a tenfold reduction of dissipation, or the film thickness can then be allowed to increase to 10^{-2} cm to maintain one-watt dissipation per joint. It may be necessary to bleed liquid helium from the supply directly to the joints to keep them at 4°K. Since superconducting joints of 10^{-9}-Ω resistance have been made in high-field superconducting magnets, the normal joints considered here may be too conservative.

It is also necessary to consider the steady electromagnetic forces acting on the conductors. These forces are large. They can be approximated (at 0.5×10^6 A) by a uniform external pressure on each cable of 400 atm (6000 psi) plus a force of repulsion of 10^9 dyn/cm (10^6 newtons per meter). Since the strength of plastics at low temperature is 10^{10} dyn/cm^2 or more, this force of repulsion can be readily supported by allowing the plastic-insulated cables to squeeze against the enclosing 4°K shell, which itself could be made of aluminum alloy or stainless steel some 2 mm in wall thickness. This wall thickness is sufficient to withstand the force.

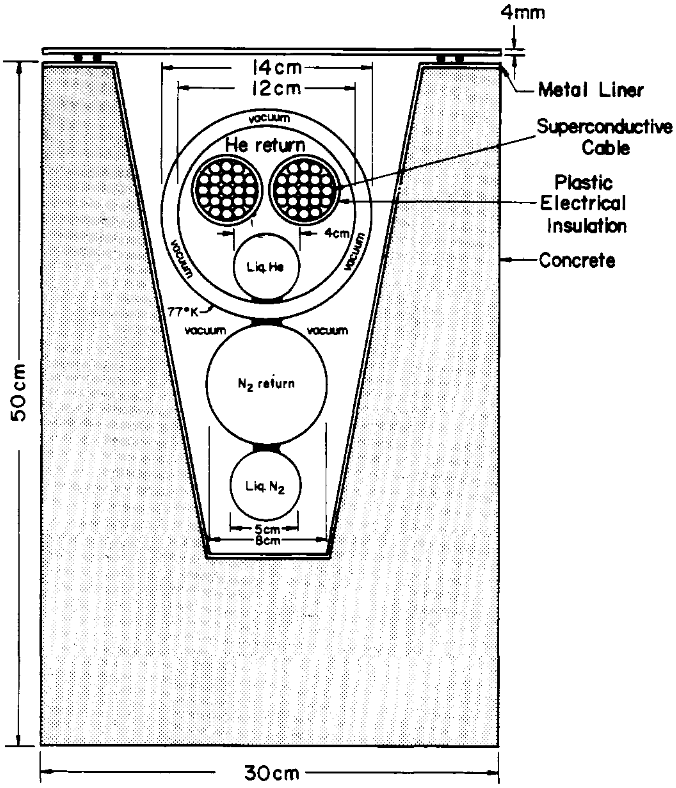

Fig. 1. Cross section of the 100-GW line.

B. Refrigeration System

A major problem of a superconducting transmission line is the refrigeration system. We shall discuss the design evolved and the reasons for the choices made in this section, and present the detailed calculation of heat leaks, the required refrigeration capacity, the flow rates of the cryogenic fluids, and the necessary vacuum pumping capacity in Appendix D.

Figure 1 shows a section of the entire transmission line and particularly the cooling system. The portion of the line which is at 4°K is interior to the 4°K vacuum wall. This pipe contains the liquid He line, as shown, and the two plastic-insulated superconducting cables. These cables may be pulled into the pipe loosely or they may be supported occasionally or continuously. The remainder of the space inside this 4°K wall serves as the He gas return (at one atmosphere).

Surrounding this wall is the 77°K radiation shield, which is cooled by the liquid (and gas) N_2 line; this, in turn, together with the gaseous N_2 return line (at 10-atm pressure) is soldered to the sheet-metal radiation shield. The shield is slit at the top so that it may be easily separated, should access to the inside become necessary. The shield-pipe assembly might be made in fifty-foot sections and the sections then soldered together, or the shield could be formed from a "continuous" coil at the installation site.

The entire transmission line rests on polyethylene, nylon, or other cheap plastic bridges, which are spaced approximately three meters apart. These supports contribute negligible heat leaks.

Finally, there is the vacuum-tight metal trough in a concrete channel which forms the basic unit of support. The concrete channel, which will probably be fabricated at the site, can be either below or above ground. The metal (tin-plated steel, or stainless steel, or perhaps aluminum) is glued with epoxy resin to the concrete. (The metal liner or a foil insert therein acts as an additional heat shield to radiation.) The metal sections of the trough are soldered together (or bonded with epoxy resin and the joints smoothed) so that they are vacuum-tight. To seal the trough, a continuous coil of aluminum (4 mm thick in 5-km sections which weigh 15 tons) is laid on top. Rubber gaskets and O-rings are used to make it vacuum-tight. This cover may be tipped up locally for inspection and repair. It has periodic apertures for vacuum pumping lines.

Looking now along the length of the line, the various elements of the cooling system and their spacing are shown in Fig. 2. Every 20 km along the line there are refrigeration stations which supply liquid He and liquid N_2. The cryogenic fluids are pumped along and flow through their respec-

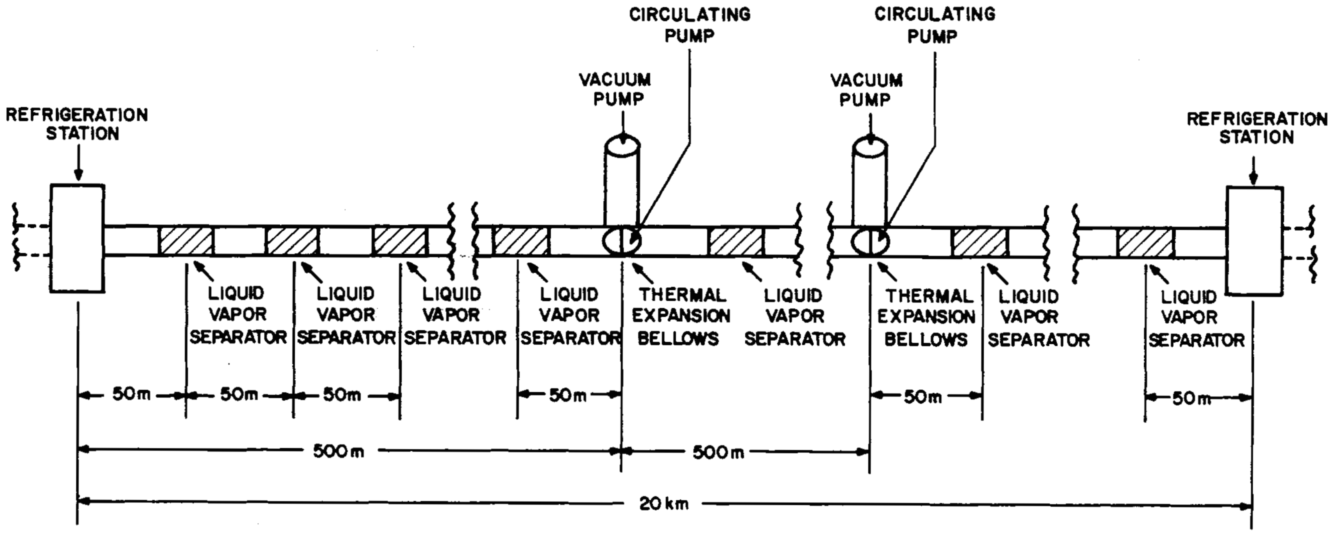

Fig. 2. A 20-km module of the 1000-km, 100-GW line.

tive pipes. It is not a minor problem, however, to cool 20 km of line from a central source. The difficulty arises because the cryogenic fluid vaporizes as it flows along the pipe. The vapor thus produced reduces the density of the mixed phase and increases the velocity required to maintain the same mass-flow rate, thus necessitating large pumping power. Indeed, even with increased vapor density at low temperatures, the effect is still prohibitive at both He and N_2 temperatures.

We solve this problem by use of liquid-vapor separators at 50-m intervals; i.e., floats which vent the gas to the gas-return manifold, while retaining liquid in the supply pipe. The liquid-vapor separators at 50-m intervals also avoid the usual problem of choking on initial cooldown of the line [9].

The separators must be capable of handling an evaporation rate of ~ 1 cm^3 of liquid He per second (or 10 cm^3/s of gas) and ~ 3 cm^3/s of N_2 liquid (or ~ 60 cm^3/s of gas). On cooldown, however, the entire liquid flow rate of the line must be allowed through the separator; clearly, it must be a device whose average position varies considerably but only slowly with demand. Separator design will not be considered further, except for noting that, if necessary, separators may include floats, switches, and motors and that they should fail-safe by closing.

Every 500 meters there is a propeller-type booster circulatory pump in the liquid He line. These are necessary because flow friction would produce a pressure drop of more than one atmosphere over the 20-km distance between refrigeration stations (at the design flow velocity of some 30 cm/s). With the booster pumps, the required pressure rise is ~ 0.10 atmosphere at a design flow rate of 0.5 liter per second.

Because of the longitudinal contraction of the refrigeration system (piping, lines, etc.) on cooldown, it is necessary to insert thermal-contraction bellows 1.5 m long every 500 m. (Thermal contraction problems are discussed in greater detail in Appendix E.)

The remaining major components of the line are the vacuum pumping stations which are spaced every 500 m along the length of the line. The vacuum pumping problem is really twofold. It is necessary to pump out the line on cooldown in a reasonable length of time (a few hours), which requires comparatively high-speed pumps (5 liters per second every 500 m), i.e., mechanical pumps. However, once the line is cooled the pumping requirements change. The cold He line is an excellent trap for all gases except He, so that only a low-speed ion pump is needed to pump out any small amount of He which may leak in (Appendix D). Indeed, the mechanical pumps may be replaced by liquid nitrogen-cooled zeolite traps, which will pump out the air which initially fills the trough.

Finally, the line as designed has a heat leak of 4×10^{-4} W/cm into the 4°K section, whidh requires a refrigeration plant using motor power totaling 0.3 MW per 20-km section. The heat leak into the nitrogen shield will be 10^{-1} W/cm and requires a refrigeration motor capacity of 1.2 MW/20 km.

These are capacities required to keep the line at 4°K. To cool the line initially in a reasonable length of time, additional capacity is required. Thus, we propose a permanently installed He refrigeration motor capacity of 1.5 MW and, if necessary, the trucking in of additional nitrogen refrigeration for cooldown. Thus, at each of the 20-km refrigeration stations we have a total installed motor power of ~ 3 MW.

To run the refrigerators, we tap power from the line. To provide 3 MW of power, we must plan to draw 15 amperes dc from the line at each 20 km station. The use of 200-kV dc motors is possible but certainly undesirable. Thus, the problem is to draw 15 amperes and convert it with high efficiency and low cost to 60-Hz ac. Since only some 200 MW total is needed for all refrigeration along the line (0.1 percent of ac-dc-ac conversion load for the *transmitted* power), the cost of conversion for refrigeration is negligible, even in comparison with the cost of the conversion equipment for the transmitted power.

Each tap itself is a source of ~ 0.15-watt heat leak into the line for which no special provision need be made.

C. Interconnection and Reliability

We shall discuss in this section the interface problems caused by the low temperature and the necessary direct-current transmission of the line. Since the line must be connected to the room-temperature alternating-current power network, we must consider how this interconnection is to be made and how to solve the problems which arise.

One of the problems involves the heat leak at the ends of the line. Wires of optimum size extending from 4°K to 77°K dissipate and conduct $\sim 10^{-2}$ watts to the helium when one ampere is being removed and returned on a pair of leads [10]. (Heavier wires conduct more heat to the cold line; thinner wires dissipate more heat.) Since 0.5×10^6 amperes are being withdrawn, the minimum heat leak is ~ 5 kW. The additional refrigeration required will cost us an investment of ~ 1.5 MW of refrigeration motor. The expense is not large, but the cooling must be good enough to prevent the heating of the superconductor and the resulting spread of normal phase down the line. There are several ways to overcome this problem. The cable can be unstranded and the junction spread over 10-m length. This will reduce the cooling required to 5 W/cm which seems feasible on a width of ~ 1 meter. There is no doubt that this can be done at reasonable cost. The current density in each of the strands is $< 10^5$ A/cm^2 and the field will be $\ll 10^5$ Oe; in any case, much below the critical values of Nb_3Sn.

Another problem is conversion of ac to dc at the source end and of dc to ac at the receiving end of the line. The detailed manner of rectification and inversion depends upon the cost of alternatives. In any case, it is likely that 6-phase or possibly even 12-phase rectification will be used, primarily to keep the ripple low. In fact, if dissipation due to hysteresis[4] is to be less than 10 percent of the normal heat leak, we must have, in a solid superconductor, a ripple current less than 450 A. With 6-phase rectification the ripple voltage is 8×10^3 volts (rms) [11], so that the nonresonant current into $\sim 100\ \Omega$ is 80 A (rms). With 12-phase rectification it is only 20 A (rms); thus the inductance of the line itself serves to reduce the ripple current. Depending upon cost, various compensating current or rectification schemes can be used to reduce the ripple to a negligible value.

The rectifiers (and inverters) themselves are likely to be mercury-arc pool type [12] or possibly silicon rectifiers and silicon-controlled rectifiers.

Finally, since it is foolhardy to assume perfect reliability, it would be wise to have a second line nearby with frequent interconnections between the lines as protection against power interruption. Connections are expensive, so relatively few are desirable. On the other hand, if no interconnections are made, there may be simultaneous breakdown of both lines. We have not definitely resolved this question; the problem is to be able to isolate a defective section of line so that it may be warmed to room temperature and repaired, which requires that there be no voltage across the lines. Too much refrigeration capacity would be required to bring the line to room temperature at every 20-km station at all times.

One solution would involve the construction of a "conventional" 200-kV, 100-GW dc line either underground or above ground near the superconducting line, with potential interconnections every 20 or 60 km. The normal line might be designed to dissipate, say, 20 percent of the transmitted power in a *single 20-km* run, for the few days required to repair a fault in a single 20-km module of the superconducting line. Routing around a fault might involve shutting off the power for a few seconds to allow automatic unsoldering of the offending section, making contact with the parallel 20-km section of normal aluminum line (thus incurring a heat leak of some 5 kw at each end of the damaged section) and switching on power as demanded, up to the remaining 80 percent of the superconducting line capacity. The switches required are not operable under load. The individual aluminum conductors required for the normal line are some 150 cm^2 in area, and the whole normal line may cost some \$80 million for conductors—a factor 10 or more less than a normal line designed to carry the full 100 GW continuously for the full 1000 km.[5] It is clear that there is still room for optimization of this emergency line, its tap spacings, etc. One use of the emergency line is to supply refrigerator and vacuum pump power initially while the superconducting line is still normal..

There are hazards involved in lines carrying such large currents, although they need not be serious. The energy in the line's magnetic field

$$W = \frac{1}{2} LI^2 = \int \frac{B^2}{8\pi} dv$$

is that stored in 1.4×10^5 gauss and 1×10^9 cm^3 or 10^{11} joules; that is, approximately the energy in 20 tons of TNT or in two tons of gasoline. In case of a ground fault, one can probably do much better than to isolate the line and to let the energy dissipate in the fault, thus gently blowing up a few feet of line. Even a dead short-circuit across a perfectly-conducting line will receive only $2\ V/z_0 = (4 \times 10^5)/10^2 = 4000$ A until the reflected waves are received from the terminals. If one short-circuits the line at some point, the current will thus rise in steps of 4000 A at a mean rate of 400 000 A/s. The resultant arc can hardly be called catastrophic. Indeed, even if the line conductors themselves are open-circuited, the flashover will short-circuit the line and there will be time for a switch at each refrigerator station to throw $\sim 50\ \Omega$ across the line. This will drain the energy in 0.1 second. Since this places 2×10^9 joules into each resistor, the temperature of each 10-ton cast-iron resistor will rise to 500°C.

D. Cost Estimate (Zeroth Approximation)

Here we try to estimate the capital and operating costs of one of the superconducting transmission lines described

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[4] In copper-covered cable the losses are the eddy-current losses in copper. For losses to be 10 percent of heat leak, ripple current must be less than 50 A.

[5] F. A. Otter, Jr., suggested (private communication, May 24, 1966) that this normal line might be made cheaply in this way.

TABLE I

Item	Estimated Cost (millions)	Basis for Estimate
Superconductor	\$550	10^4 tons of Nb_3Sn @ \$26/lb*
Refrigeration along line	\$ 25	\$0.5 million per He refrigerator of 1 kW at 4 K every 20 km†
End refrigeration	\$ 5	These must handle a total of 10 kW (2 × 5 kW) or an equivalent of 10 1-kW refrigerators at \$0.5 million each
Vacuum pumps	\$ 1	Assumed \$500 per pumping station
Fabircated metal	\$ 20	Assumed cost of \$1/lb and weight of 100 g/cm
Concrete	\$ 5	Assumed cost of \$10/yd^3 with total volume of $\frac{1}{2}$ yd^2 × 10^3 km
Converters	\$200	Cost of mercury-pool rectifiers ~\$1/kW‡ and of silicon rectifiers and silicon-controlled rectifiers ~\$1 kW§
Total	\$806 (Million)	

* Ultimate cost of Nb_3Sn estimated from Nb cost of \$36/lb (*Chemical and Engineering News*, p. 65, February 3, 1964) and Sn cost of \$2/lb (*Yearbook of the American Bureau of Metal Statistics for 1962*, New York, 1963 p. 132).

† Informal quotation to IBM Corporation by Arthur D. Little, Inc., Cambridge, Mass., February 1966.

‡ Informal quotation to IBM Corporation by General Electric Company on 3-MW mercury-pool rectifier (20 kV, 150 A).

§ Informal quotation to IBM Corporation by International Rectifier Corporation on silicon rectifier [1.2 kV, 550 A (rms)] and silicon-controlled-rectifier [1.2 kV, 400 A (rms)].

above. We shall not include in the cost estimate such items as the cost of right-of-way acquisition and clearing (although this cost is likely to be an order of magnitude less than that for conventional overhead line), cost of on-site labor, etc. In addition, we do not include the cost of transformers at the terminals, since these are required on any conventional line, no matter how short.

A last item which must be considered part of the capital cost of the superconducting line is the converters. There are no single converters currently in existence which are capable of handling the currents and voltages required. There are, however, mercury-pool converters which will safely handle 100 kV and a few thousand amperes [13]. If we extrapolate from the available cost figures of \$1/kW, the total cost for two stations (one at each end) is \$200 million which is not very high. (See Table I.)

Although the above costs may be in error by a significant factor, it is clear that the major cost (apart from converters) is the cost of the superconductor. There are various schemes by which the critical current density of superconductors might be increased. One such scheme is to fabricate synthetic high-field, high-current superconductors with small superconducting filaments in a suitable normal metal matrix [14]. The matrix-metal-to-superconductor-volume ratio, as well as the filament diameter and the materials, have to be appropriately chosen, otherwise no high-field properties are obtained [15]. Another scheme has been demonstrated by Bean et al. [16]. They have shown that current densities greater than 10^6 A/cm^2 (with 10^7 A/cm^2 conceivable) can be obtained (in V_3Si) by introduction of defects by internal fission in the superconductor. If, for example, the critical current density could be increased to 10^7 A/cm^2, a hundred-fold reduction in the cost of the superconductor would result.

The operating costs arise primarily from the energy required to liquefy He, N_2 and the power required to run the vacuum pumps. In normal operation the total He refrigerator-motor power is 0.3 MW × 50 or 15 MW. Therefore, at a busbar cost of 6.8 mil/kWh, the He refrigerator operating cost is ~\$1 million per year. The nitrogen cost is approximately \$4 million per year, since the refrigerator capacity for the nitrogen line is roughly four times that of the He line. Thus, the operating costs are approximately \$5 million per year which, of course, compares very favorably with transmission losses on an ordinary line of this capacity.

SUMMARY AND CONCLUSIONS

We summarize the design characteristics of the line as follows.

Power capacity	100 GW (10^{11} W)
Voltage (dc)	200 kV (2 × 10^5 V)
Current (dc)	0.5 × 10^6 A
Line temperature	4.2°K (liquid helium)
Radiation shield	77°K (liquid nitrogen)
Length of line	1000 km
Refrigerator spacing	20 km
Gas-liquid separator spacing	50 m
Booster pump spacing	500 m
Vacuum pump spacing	500 m
Thermal expansion bellows 1.5 m long (superconductors wound helically) spacing	500 m
Fraction of power dissipated in line and leads	$<10^{-7}$
Fraction of power used for refrigeration	$<10^{-3}$

We have offered another solution to the problem of economical electrical energy transmission, by sketching a design for a large-capacity, long-distance superconducting line and estimating the capital and operating costs for such a line. If our cost estimates are not too much in error, it is clear that the most economical solution to the over-simplified power transmission problem posed in the Introduction is a superconducting line of the general design described. This becomes particularly apparent when annual costs are examined. Thus, coal transportation cost is approximately \$1 billion a year, ordinary EHV transmission losses ~\$340 million a year, while the superconducting-line "losses" amount to only ~\$5 million a year. Even the capital costs may favor the superconducting line over conventional EHV. The capital investment in EHV transmission is ~\$1 to \$1.5 billion, whereas the superconducting line cost is ~\$606 million. (The converter costs are the same for EHV dc and superconducting line and therefore have not been included in the comparison.)

The superconducting line has essentially fixed annual operating costs; i.e., the refrigeration cost is almost independent of the current-carrying capacity of the line. Also, the capital costs associated with the refrigeration system are the same regardless of line capacity (assuming fixed 4.2°K operating temperature). What does scale is the superconductor cost (and converter cost) which varies directly with the power capacity of the line. With ordinary EHV transmission, as the power capacity is reduced there comes a point at which ac transmission becomes practical (eliminating converters). Losses then scale with power level, as does the capital investment in the line.

Thus, for sufficiently low power levels and over sufficiently short transmission distances, it will undoubtedly be more economical to use conventional ac EHV transmission. However, there will exist a power level and distance beyond which superconducting lines will prove more economical. Clearly, detailed engineering design and cost analysis is necessary to determine exact cross-over points. We have shown, however, that circumstances may be such as to favor the novel approach of a superconducting power line.

APPENDIX A
CONVENTIONAL EHV TRANSMISSION

Extra-high-voltage (EHV) transmission is presently carried out at 345 and 500 kV with, however, comparatively low capacity (say 1.2 GW).[6] If 100 GW were to be transmitted in the conventional way, the transmission must be dc and converter costs would be the same as in the superconducting case.

To carry the current one could use two aluminum (for the sake of argument consider aluminum rather than ACSR) conductors ~ 1200 cm^2 each in area [17] and at much higher voltage. The resistance of such a line (assuming 60°C operating temperature) is $\sim 0.3\ \Omega$, which results in ~ 5 percent dissipation. To carry 100 GW a distance of 10^3 km at 5 percent dissipation would require a current of 1.4×10^5 A at 750 kV.

Since there exist no detailed cost estimates in this power range, we obtain a rough lower limit to the costs involved as follows.

The weight of the aluminum conductors is $\sim 8 \times 10^5$ tons, which at \$0.25/lb [18] for aluminum (or \$0.38/lb for steel-reinforced cable) is a cost of \$440 million for the material or some \$670 million for the conductors. The cost of poles. etc., is probably several times this amount for a total capital investment of $\sim$\$1 billion.

The main point, however, is that the line is extracting an annual toll of 5 percent on the electricity transported, or a loss of $\sim 5 \times 10^{10}$ kWh per year at an annual cost of \$340 million.

APPENDIX B
SUMMARY OF PROPERTIES OF SUPERCONDUCTORS PERTINENT TO CURRENT-CARRYING CAPACITY

Superconductors are materials which below a temperature T_c (called the transition temperature) have zero electrical resistivity and exclude magnetic fields. This is true as long as the current density within the superconductor remains below a critical value and as long as applied magnetic fields are sufficiently small. Thus, provided these critical variables (temperature, current density, and magnetic field) have sufficiently low values, the resistance of the superconductor is zero. There are two kinds of superconductors, called Type I and Type II.[7] They are distinguished by their rather different magnetic and current carrying behavior (see Fig. 3).

Weak magnetic fields are excluded from the interior of both Type I and Type II superconductors (except from a small surface layer, typically on the order of 500 Å). As the externally applied magnetic field is increased, in the case of Type I superconductors, the field penetrates completely at the thermodynamic critical field, and at higher fields the behavior is much the same as that of any normal metal.

In Type II materials, field penetration begins below the thermodynamic critical field (although this field may be much higher than the corresponding field in Type I materials). The field penetration continues with increasing field, to fields as high as 200 kG in Nb_3Sn. It is extremely important to note that a large fraction of the superconductor remains superconducting in high fields, although the magnetic field, on a macroscopic scale, penetrates uniformly [Fig. 3(a)].

Another crucial difference between Type I and Type II superconductors is their current-carrying behavior. In Type I any transport current is carried on the surface (a consequence of flux exclusion) and, therefore, the total current-carrying capacity increases only as the diameter of the conductor. On the other hand, in Type II superconductors, in a sufficiently high field so that the field has penetrated, the current density is essentially uniform over the cross section and thus the total current is proportional to the square of the diameter. It should be pointed out that the achievement of high-field high-current capability in these superconductors actually depends critically upon the existence of defects in the material which act as pinning sites to flux lines, and prevent their motion under the influence of Lorentz force ($J \times B$). When it does occur, the motion of flux lines produces a voltage drop along the length of the superconductor and thus effectively introduces resistance.

APPENDIX C
LOSSES IN ALTERNATING-CURRENT TRANSMISSION

Hysteresis Losses

It would be convenient to transmit conventional 60-Hz ac, for numerous reasons, such as ease in tapping power and so forth. This is not possible for several reasons, two main ones being hysteresis losses in Type II superconductors (in fields greater than H_{c_1}; in practice, fields will always be greater than H_{c_1}), and anelastic losses produced by the changing magnetic field.

The hysteresis losses are due to the irreversible motion of flux under alternating-current conditions. In recent work Bean et al. [19] have shown experimentally that hysteretic

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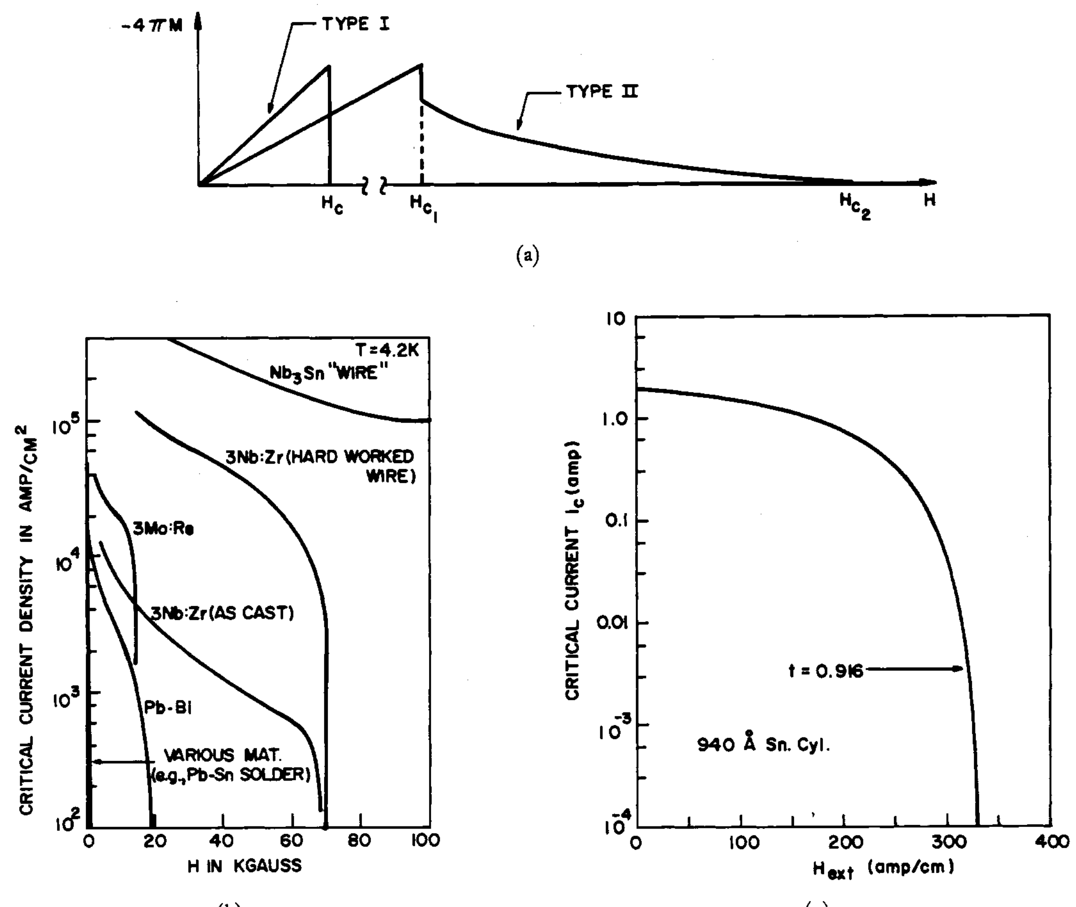
[6] A 2-GW, 266-kV *underground* dc system some 85 km long is under contract to be built into London (*N. Y. Times*, April 30, 1966).

[7] For a good review of Type II superconductors see, for example, J. E. Kunzler, "High-field superconductivity," *Materials Research and Standards*, pp. 161–171, April 1965.



(a)

(b)

(c)

Fig. 3. (a) Magnetization vs. field for Type I and Type II superconductors. (b) Critical current density vs. field for various Type II superconductors. [After J. E. Kunzler, "High-field superconductivity," *Materials Research and Standards*, pp. 161–171, April 1965.] (c) Critical current vs. field for a Type I superconductor (a cylindrical thin film sample of tin, 940 Å thick). [After J. Mydosh and H. Meissner, *Phys. Rev.*, vol. 140, p. A1574, 1965.]

loss (which is constant per cycle) is indeed the loss mechanism at power frequencies.

The hysteresis loss in a two-conductor superconducting cable carrying a large alternating current of amplitude I_0 is (Bean [19], p. 19)

$$H = 4 \times 10^{-9} I_0^2 F \quad \text{W/cm} \tag{1}$$

where F is the frequency in Hz. Thus, if transmission were to be 60 Hz the hysteresis loss would be approximately 6×10^4 W/cm!

Under noncyclic conditions there is also loss. Thus, we must consider the energy loss on energizing of the line. Bean [19] calculates this energy dissipation (per cable) as

$$0.5 \times 10^{-9} I_0^2 \quad \text{J/cm}$$

or a power loss of

$$H = 10^{-9} I_0^2 \tau^{-1} \quad \text{W/cm} \tag{2}$$

where τ is the line-energizing time, i.e., the time to bring the line-current (dc) from zero to a value I_0. For a τ of one hour, the loss is 0.07 W/cm; for a τ of one day, however, the loss is 3×10^{-3} W/cm.

Finally, because the current will not be ripple-free (converter ripple and fluctuating demand), we need to know the loss for small-amplitude alternating currents (ΔI).[8] Again Bean [19] gives this as

$$H = \frac{4 \times 10^{-10} (\Delta I)^3 F}{J_c R^2} \quad \text{W/cm} \tag{3}$$

where J_c is the critical current of the superconductor and R is the radius of the cable. For $J_c = 10^5$ A

$$H = 2 \times 10^{-15} (\Delta I)^3 F \quad \text{W/cm.} \tag{4}$$

Anelastic Losses

The magnetic field squeezes the wire with pressure, $P(=B^2/8\pi)$. For a wire of modulus of elasticity Y, the stored energy is

$$\varepsilon = \frac{P^2}{2Y} \pi R^2 \text{ erg/cm.} \tag{5}$$

In most plastics and even metals at ordinary temperature only one-half of this energy is recovered; i.e., the anelasticity a is 1/2. At low temperatures a will be smaller unless there is slippage.

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[8] Note that if the cable is stabilized by the inclusion of a copper sheath, these ripple currents will flow in the copper sheath and the losses will be normal eddy-current losses.

$$\varepsilon \cong \frac{(4/\sqrt{2} \times 10^8)^2}{2 \times 10^{12}} 5 \cong 2.5 \times 10^5 \text{ erg/cm}.$$

Since the stress goes to zero 120 times per second the dissipation per centimeter is

$$h = 3 \times 10^7 \, a \text{ erg/cm}\cdot\text{sec} = 3 \, \boldsymbol{a} \text{ W/cm}. \tag{6}$$

To have this loss comparable with the heat leak requires that $\boldsymbol{a} < 10^{-5}$.

These problems are avoided by dc transmission, as is the hysteresis loss in hard superconductors.

APPENDIX D

REFRIGERATION SYSTEM

The heat transport into the line will be primarily by radiation. (Heat influx from 300°K to 77°K by conduction through nylon bridges of approximately 4 cm^2 in contact area and 8 cm long is roughly 10^{-3} W/cm or 1 percent of radiative leak; from 77°K to 4°K for 1 cm^2 in contact area and 2 cm in length is approximately 10^{-5} W/cm again for nylon spacers 3 m apart, assuming a $\bar{\lambda} = 10^{-4}$ W/cm · °K over this temperature range.) The radiation per unit time per square centimeter between two surfaces of emissivity ε is given by

$$\frac{\varepsilon}{2} \sigma (T_1^4 - T_2^4)$$

where T_1 and T_2 are the temperatures of the surfaces and σ is Stefan's constant (5.67×10^{-12} W/cm$^2 \cdot$deg^4). An emissivity of 0.05 is easy to achieve.

A radiation shield at 77°K is desirable (and conventional) to interrupt radiant heat at this temperature (T_2) and allow it to be rejected at room temperature (T_1) by expenditure of W units of work (Carnot cycle)

$$W = \frac{T_1 - T_2}{T_2} \frac{H}{E} \tag{7}$$

where

H = heat influx

E = refrigerator efficiency; i.e., fraction of ideal thermodynamic efficiency (~0.5 at 77°K but only ~0.2 at 4°K).

The shield diameter is ~14 cm, so the heat influx into the nitrogen shield is ~10^{-1} W/cm, and the heat flux from 77°K to 4°K (diameter ~12 cm, ε still 0.05) is 4×10^{-4} W/cm.

Therefore, over a 20-km length there is 2×10^5 W to be rejected from 77°K and 800 W from 4°K. The corresponding refrigerator motor capacity must thus be 1.2 MW for 77°K and 0.3 MW for 4°K (working to 300°K). This motor capacity is required to keep the line at 4°K. However, ~1.5 MW of installed 4°K refrigerator motor capacity is necessary to cool the line in several days from 77°K to 4°K.

It will require an even longer period of time to cool to 77°K with available capacity unless portable refrigerators to increase the total capacity are trucked in.

We now discuss the refrigeration problem in detail, and consider a pipe of radius R carrying fluid of density ρ at temperature T. A steady distributed heat flow, h erg/cm · s boils away liquid which is vented continuously into the return manifold. Consider the radius R independent of distance x, and the overall length of the line as L. Denote the flow velocity as v. The velocity v, of course, varies along the length of the pipe and is zero at the far end ($x = L$). The volume of liquid transported per unit time is $V(x) = \pi R^2 v(x)$ and

$$\frac{dV(x)}{dx} = -\frac{1}{\lambda}\left[h + \pi R^2 v \frac{\rho v^2}{2} \frac{1}{100R}\right] \tag{8}$$

where λ is the heat of vaporization per cubic centimeter.

The first term on the right is the heat leak and the second term is the friction work. Since the Reynolds number for He in this (smooth) pipe is 5×10^5, the flow is clearly turbulent and the friction coefficient is ~0.0033 (velocity ~30 cm/s). Therefore, the kinetic energy of flow is lost as heat approximately every 50 diameters, and the second term of (8) represents this approximation.

On this basis we derive upper limits on the interval between refrigerators, and choose a distance well within the feasible range. Equation (8) can be written

$$-\lambda \pi R^2 \frac{dv}{dx} = h + \frac{\pi R \rho v^3}{200} \tag{9}$$

or

$$-dx \frac{h}{\pi R^2 \lambda} = \frac{dv}{1 + \dfrac{\pi R \rho v^3}{200h}}. \tag{10}$$

Defining $\pi R \rho / 200h = \beta^{-3}$, the integral of (10) becomes

$$\frac{hL}{\pi R^2 \lambda} = \beta/\sqrt{3}\,\pi/6 + \beta/3 \left[1/2 \ln \frac{[\beta + v(0)]^2}{\beta^2 - \beta v(0) + v^2(0)} + \sqrt{3} \tan^{-1} \frac{2v(0) - \beta}{\beta\sqrt{3}}\right]. \tag{11}$$

The limiting length (as $v(0) \rightarrow \infty$) is

$$L_{\max} = \frac{\pi R^2 \lambda}{h} \frac{2\pi}{3} \beta/\sqrt{3}. \tag{12}$$

A practical maximum, however, is 1/2 $L_{\max}$ or, say,

$$L = \frac{R^2 \lambda \beta}{h} \sim 4\lambda R^{5/3} \rho^{-1/3} h^{-2/3}.$$

The corresponding initial fluid velocity is ~1/3 β.

Consider the N_2 problem. We have calculated $h \sim 10^6$ erg/s · cm, $\rho \sim 1$ g/cm^3, and $\lambda = 1.6 \times 10^9$ erg/cm^3. Therefore, for the 5-cm diameter N_2 pipe $L_{N_2} < 30$ km. For He refrigeration $h \sim 4 \times 10^3$ erg/cm · s, $\rho = 0.125$ g/cm^3. For the 4-cm diameter He pipe we then have $L_{\text{He}} < 28$ km. Thus we might well choose a refrigerator spacing of 20 km.

The refrigeration rate is $\pi R^2 v \lambda = \pi R^2 \lambda (1/3\ \beta)$ or approximately 1 kW at 4°K. Larger flow rates can be obtained during cooldown from the same refrigerator.

Little energy is lost in allowing the N_2 vapor to warm to 300°K on its return to the refrigerator, since the latent heat of vaporization is roughly equal to the energy required to cool to the boiling point.

With helium, however, precisely the opposite is true, since the latent heat per cubic centimeter is smaller by a factor ~60; i.e., most of the energy required to liquefy helium gas goes into cooling the gas to 4°K.

The N_2 evaporation rate is ~2 liters per second in 20 km or a gas flow rate at STP of 3000 cfm. We propose to operate at a pressure of 10 atmospheres to decrease the velocity by a factor of ten and to reduce the pumping power.

For He the critical pressure is 2.26 atmospheres; thus, not much over-pressure is permissible. However, the vapor density at 4°K (1 atm) is about 10 per cent of the liquid density. A return pipe at 4°K and ten times the area will then not add much pressure drop and will only double the helium investment (~28 million ft^3 He gas STP, costing $\$2 \times 10^6$ and ~10 percent of the annual production).

Cooling by supercooled N_2 and He^4 has been considered but was found to be much less effective. Superfluid He (He II) could be used without pumping but only over much shorter distances.

VACUUM PUMPS

The limiting pumping speed of a pipe (in cm^3/s) of diameter D and length L is [20]

$$C \sim 1.2 \times 10^4 D^3 L^{-1}.$$

The volume is $V = \pi/4 D^2 L$. Thus the pumping time constant is

$$t = \frac{V}{C} = 6 \times 10^{-5} L^2 D^{-1} \text{ seconds.}$$

Thus the pump capacity and pump separation depend upon how rapidly it is desired to pump out the air. For example, since $D = 15$ cm, if we choose $t = 200$ seconds (or total pump-down time of ~15 minutes) the pump separation must be ~50 m. The speed of the pumps then must be the volume of line in 50 meters per 200 seconds or 25 l/s. However, if we choose $t = 3$ hours (a total pump-down time of ~10 hours) the pump separation can be 500 m with a pumping speed of 5 l/s.

APPENDIX E

DIFFERENTIAL EXPANSION

The cryogenics engineer, like his steamy brother, must reckon with thermal expansion. Ordinary metals on being cooled to 0°K contract ~0.3 percent, some plastics ~2 percent. Therefore, upon cooling, a long line fixed at the ends is subjected to a 0.3 percent longitudinal stretch which, with an elastic modulus $\sim 10^{12}$ dyn/cm^2 corresponds to a tension ~3000 atm or 50 000 psi. It is desirable to avoid such stresses in order to eliminate the massive insulating clamps and additional gear required to stretch the line, and in particular to avoid the accompanying heat conduction. It is not impossible to use such techniques, particularly with low-thermal-expansion materials, but it is desirable to have the possibility of using any reasonable materials in the line. A contraction ~0.3 percent on a 20-km run amounts to ~60 m, which is large. Fortunately, however, it is not necessary to allow for differential expansion between the parts of the structure which are at 4°K and 77°K unless very different materials are involved. (Even when the materials do have vastly different coefficients of thermal expansion this part of the problem can be solved by the frequent insertion of bellows in the N_2 lines of say 0.1 percent or ~50 cm every 500 meters.) Undoubtedly the simplest solution to the problem of the contraction of the cable itself is to twist the superconducting cables as they are fed into the housing. A twist which increases the length by a few percent will allow (preferably with a compressible center) an extension of 0.5 percent without much cost penalty. Finally, to take care of the absolute contraction of the large-diameter housing for the cables and for the helium line, bellows, ~1.5 m long every 500 meters, can be inserted.

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