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ELECTRONICS FOR THE IBM GRAVITY WAVE DETECTOR--CONCEPT, IMPLEMENTATION, AND EXPERIENCE

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Summary

Our apparatus--antenna, transducer, signal processor, and calibrator--was designed to settle the question of the existence of gravity waves at 1.7 kHz of the numbers and intensities claimed at the time we began (end-1971). Thus our design criteria were: (1) modest sensitivity, (2) sensitivity independent of signal arrival time and state of excitation of the antenna, (3) absolute calibration with pulsed mechanical excitation of the antenna, (4) full simulation of the apparatus, (5) hands-off computer analysis with every point published.

We recognized that a single bar would ultimately be limited by some Boltzmann distribution of noise at sub-thermal temperature (18 K for our bar at 300 K), and that such an Ideal antenna would be equivalent to an ideal coincidence pair of antennas, each of half the mass. Transducer, amplifier, signal processor, and programming were all done by the experimenters in order to reduce the cycle time for introducing Improvements. Before the antenna and amplifier were ready, the processing algorithms were developed and tested with digitally-simulated antenna output, and many problems avoided. Any excess local noise proved to be sufficiently infrequent so that the single antenna could negate claims by Weber of the detection of gravitational radiation.¹ The computer processing obviated the need for temperature control of the antenna or for tracking of the bar resonant frequency with the reference oscillator.

Introduction

General relativity allows one to calculate the radiation of gravitational energy by accelerated masses (or more precisely, by a time-varying mass quadrupole). But the efficiency of radiation from slowly moving objects is small, and the coupling with masses which might be used as detectors is also small. Thus the few events per day published by Weber and Interpreted as pulses of gravitational radiation, if they were of galactic origin, each corresponded to the transformation of a large fraction of a solar mass into gravitational radiation, leading to the disappearance of much of the mass of our galaxy over a period of 10⁸ years, whereas all other evidence points to an age on the order of 10^{10} years. Thus there was great interest among theorists in this problem of the large-scale structure of matter and its coupling with gravity. It was similarly of interest to see whether the observations were sound.

Greatest importance lay not in verifying the details of some calculation of gravitational radiation, but rather in determining whether "large pulses" of gravity waves ("GW") were incident on earth several times per day.

Detection of GW requires consideration of several processes, only some of which are under the control of the experimenter:

1	1. source of GW
	2. transmission to earth
	excite GW antenna
	4. energy transfer to transducer
	amplify transducer signals
	reduce redundancy
	record without loss of
	significant data
	 efficient computer search
	for gravity wave signals

In step (3), the antenna is excited not only by gravity waves (if they are incident) but by thermal excitation and by local vibration and acoustic noise. Similarly, in step (5) the amplification of the transducer signal is afflicted with thermal and non-thermal circuit noise. Our results are summarized in

several papers.^{2,3,4,5} In a word we showed (as have others) that the detections reported by Weber were not due to pulses of gravitational radiation, and, in some cases, not to any phenomenon of physical interest.

In our published papers, we have analyzed in considerable detail the noise and the detection efficiency of Weber's electronics, in order to be able to compare the sensitivity of our apparatus with his. We shall not repeat those analyses here, confining ourself to discussion of electronics of our own experiment.

Concept

Our gravity wave antenna proper is a bar of aluminum alloy type 7075-0 150-cm long by 38-cm diameter and weighing 480 kg. The lowest longitudinal compressional mode has F_B=1637 Hz. The bar is operating in a vacuum of less than 0.2 Torr in a normal laboratory. It is supported, axis horizontal and oriented east-west, by a steel cable from a three-stage mechanical filter of 50-kg cast-iron masses separated by rubber vibration isolators. The vacuum chamber with its contents is further isolated at low frequencies by suspension from a pneumatic servo isolation frame.

The amplitude of vibration of the resonant bar is measured by a transducer coupled to the end of the bar. A piezoelectric ceramic cube a few millimeters on a side (lead zirconate titdnate--PZT-4) provides a signal proportional to the displacement between the end of the aluminum bar and a selsmic mass (5 kg) which is supported by steel wires from pins set into the aluminum bar. (Fig. 1) The resonant frequency of the selsmic mass with the stiffness of the ceramic is F_x =800 Hz; so that the mass is a nearly stationary anvil from which to measure the vibration of the bar



Fig. 1. The piezoelectric ceramic transducer measures the displacement of the end of the aluminum bar with reference to the seismic mass suspended by wires from the end of the bar.

In principle, then, a pulse of gravitational radiation incident upon the at rest will excite it into bar oscillation at F_B, which oscillation will persist for the natural damping time of persist for the natural damping of the the bar $(Q/2\pi \text{ cycles to } 1/e \text{ of the clearly, whatever the clearly, whatever the clearly, whatever the clearly whatever the clearl$ Initial energy). Clearly, whatever the preexisting state of oscillation of the bar, the incidence of the pulsed gravity the same wave will yield precisely additional vector amplitude. Since the ringing time of the bar is on the order of 1.4 sec; and because the preexisting energy of the bar is in any case not zero because of the presence of thermal drawn from a Boltzmann excitation TR distribution at room temperature GW pulses can best be detected by means which essentially an algorithm of subtracts from the amplitude at the end of the sampling interval the amplitude at the end of the previous sampling interval. In this way, detection of gravity waves can be made independent of the preexisting state of oscillation of the bar, and the system can have uniform sensitivity to gravitational radiation.

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The simple subtraction of vector amplitudes thus allows measurements to be made with good time resolution without Interference from preexisting excitation of the bar. It makes neither economic nor physical sense to choose an infinitesimal sampling interval τ--as nor we shall see there is an optimum for the detection of signals in the presence of thermal amplifier noise and bar damping. Finally, we shall see that an algorithm slightly more complicated than simple subtraction of vector amplitudes is desirable in order that the system have a unique response to a given gravity wave input (even in the absence of noise) independent of the arrival time of the gravity wave impulse within a sampling interval τ . We shall illustrate this point, by a brief comment on the 7 publication by Bramanti, et al.

Implementation



Fig. 2. Schematic of preamplifier mounted on seismic mass. Piezoelectric ceramic transducer of 28 pf capacitance is connected at input. Note that injudicious choice of C_L ,

together with stray capacitance C_{f} , can result in placing

across the input a noiseless resistor of magnitude $R = (1/g_m)(C_L/C_f)$, where g_m is the transconductance of the input FET. More generally, if the first op-amp has a gain G with a 90° phase lag at f_0 (before external feed-back via a resistor R_4) then the input admittance of the FET becomes

$$\frac{1}{R} = g_m \omega C_F \left(\frac{1}{\omega C_L} - \frac{R_4}{|G|} \right).$$

Thus the amplifier may damp the bar, decrease its apparent losses, or even render it unstable, depending on the seemingly irrelevant values of R_4 and G. Similar effects may have troubled other workers, especially those without facilities to inject known mechanical energies into their bars.

of a fleld-effect transistor followed by two integrated-circuit op-amps is mounted on the seismic mass, and the output (proportional to the displacement of the bar) is led via cables along the bar and its support. The signal is then fed in parallel to two phase-sensitive detectors (synchronous reversing switches) operated respectively by direct and quadrature signals from a stable oscillator of frequency $f_0 = f_B + \Delta f$; $-1 < \Delta f < 1 + z$). The detector outputs feed resettable Integrators. At the end of each Interval τ (40 cycles of f₀ or \sim 24 msec), the outputs of the Integrators are digitized (±10 V full scale) by 8-blt A/D converters yielding two 8-bit (1-byte) amplitudes which are written (plus parity bits)₆ onto an incrementing magnetic tape.⁶ The integrators are then reset.

The data are usually grouped in blocks of 16,384 bytes (3 min of elapsed time), which include 4 bytes of time information from a quartz-crystal counter. About 40 h of data can be accumulated on a single 1200-ft magnetic tape. Each data block is then processed by a computer which first computes the autocorrelation function and from it the decrement δ of the bar ($\delta = \pi \tau f_0/Q$) and its offset $f_0 - f_B$. These data are then used to predict from each pair of amplitudes [a vector v(t_n) in the phase plane] the

amplitudes of the next τ seconds later,

$$v*(t_n+\tau) \equiv v(t_n) \exp(-\delta), \qquad (1)$$

after obvious corrections for frequency offset. Predicted amplitudes are then subtracted from the measured amplitudes, with result

$$d(t_{r}) \equiv v(t_{r} + \tau) - v * (t_{r} + \tau).$$
 (2)

estimates represent The d(t_n) (corrupted by amplifier noise) of the successive amplitude changes during the Interval t by virtue of the coupling of reservoir at room the bar to the temperature (damping) and through the absorption of any gravity waves. To each of the $d(t_n)$ corresponds an energy E_n which would be given to a bar at rest by its impulsive excitation to an amplitude $d(t_n)$. If a large calibration pulse can be calculated <u>a prioring</u> to give the bar at rest an energy E_c, and if the normal computer processing as outlined above yields for the interval contairing the calibration pulse an amplitude change d_c, then the d(t_n) may be taken to represent energies

E_n≖E_c[d(t_n)/d_c]².

Figure 3 shows a typical auto-correlation function AC(mt) for the bar in thermal equilibrium at 295 K calculated from 8000 successive data points with an interval $\tau=24$ msec. The autocorrelation function may be used to compute the mean bar energy. If AC* is the extrapolation to zero delay, the average bar energy is

$$(E_B)_{avg} = [v^2(t_n)]_{avg} (E_c/d_c^2) AC*.$$
 (4)



Fig. 3. The normalized autocorrelation function $AC(m\tau)$ for x (curve a) and the orosscorrelation function for x and y (curve b) computed from the digital data of 14 March 1972. Then $AC(0)\equiv 1$, and the measure of amplifier noise is the deviation from 1 of the extrapolated value AC^* of $AC(m\tau)$ as as m + 0. Here $AC^* = 0.94$. The correlation functions oscillate with frequency $f_B - f_0$.

This measured $(E_B)_{avg}$ is used to define a bar temperature T_B , $kT_B \equiv E_B$. Thermal excitation of the bar, and noise from the amplifier, should result in E_n being Boltzmann distributed with frequency of occurrence $N=N_0exp(-E_n/kT_e)$. The value expected for T_e as a result of bar temperature and amplifier noise may now be calculated as

$$T_{a} *= 2T_{B}[\delta + (1 - AC *) / AC *].$$
 (5)

Large t allows a larger influence of bar temperature [the first term in Eq. (5)], and short τ a wider-band contribution of noise from the amplifier [the second term In Eq. (5)]. To provide sensitivity Independent of impulse arrival time interval τ, within the we use an Eq.(2), but similar to algorithm Involving

V(t_+mt)(m=-2,-1,1,2),

(3)

for which the expected effective value of

Tels

T_*=

$$2T_{B} \delta \frac{(K+1)(2K+1)}{3K} + \frac{(1-AC*)}{KAC*} \qquad (6)$$

With $\tau = 24$ msec (40 cycles), (1-AC*)/AC* =0.06; $\delta = 0.010$. T_e*=33 K, as compared with the observed 1-day averages of T_B=300.1 K and T_e=28.9 K. Thus, for our bar Q/\pi=4200. (In Eq.(6), K=2m+1.)

To provide a known oscillation energy to the bar, we use N periods of a calibrating voltage, the value of which in successive half-cycles of the bar reference oscillator f_0 is +V,0,-V,0. The energy given to a long thin bar by this signal applied to a plate of area A spaced s cm from the end of the bar is (cgs-esu)

 $E_{c} = A^{2} N^{2} V^{4} / 4 \pi M_{B} \omega_{0}^{2} S^{4}.$ (7)

We have $A=25\pi$ cm²; s=0.17 cm; N=5. In deriving Eq. (7), we make use of the fact that the oscillation energy of a long bar is related to the peak amplitude a as $E=1/2M_{\mu}\omega^2a^2$, with the effective mass of the bar $M_e=1/2M_B$. With V=10V=10/300

esu, $E_{=}4.31 \times 10^{-13}$ erg or 3130 K. The mean energy of the bar is thus determined from Eq.(4) to be 300.1 K, in reasonable agreement with the room temperature of 295 K.

significant data from the The experiment are contained in the pair of numbers $d(t_n)$ computed each τ seconds which represents the vector change in amplitude of oscillation of the bar. A typical distribution⁴ of the energies En corresponding to the observed amplitude changes is shown in Fig. 4. The straight is N=N₀exp(-E/kT_e), with an line effective temperature T_e=18.5 K. Except possibly for one pulse, the totality of is reduced thus far data the thermal from indistinguishable distribution which would be obtained in The the absence of gravity waves. mechanical against and Isolation electrical disturbances is evidently good enough to make such extraneous influences contributors the to negligible oscillation energy of the bar.

The energy scale of mined by comparison Fig. 4 is determined by with an electrostatic calibrator and are given in convenience in units of the mean kT r (T_=room oscillation energy line the straight temperature), (Boltzmann distribution) being the result expected if only thermal and amplifier Data blocks present. noise were callbrator pulses are containing processed together with regular data just like any other block, and the calibrator pulses are identified as if they were



Fig. 4. Histograms of observed impulse energies. Solid curve, Boltzmann distribution with effective temperature $T_o=18.5 \text{ K}.$

gravity waves. The slope of the line defines effective an temperature Te=0.063Tr which characterizes the experiment. The histogram Includes 4.1x10⁷ values obtained during a 27-day period (3-30 December 1973). Only one period (3-30 December 1973). measurement interval (time: 23:01 :22±1 sec, E.S.T. 12 December 1973) contained a pulse substantially above the noise. We will discuss this pulse below. We have obtained the efficiency for detecting pulses of energy ${\rm E}_{\rm g}$ above a

threshold value E_t , both theoretically and experimentally. Let a large number of such pulses occur. In the presence of Boltzmann-distributed noise characterized by mean value kT_e , the detection algorithm will return a distribution of apparent pulse energies E, related to the Rician distribution function;

 $F(E) = (1/kT_e) exp[-(E+E_p)/kT_e]I_0(X),$ (8)

 $X = [2(EE_g)^{1/2}]kT_e'$

where I₀(X) is the modified Bessel function of zero order. Expanding, we find

$$F(E) \simeq [1/(4\pi E_g kT_e)^{1/2}]$$

Xexp[-(E_g+kT_e-E)^2]/4E_g kT_e, (9)

 $|E-E_g| \langle \langle E_g, E_g \rangle \rangle kT_e$.

Equation (9) is plotted in Fig. 5(a) for $E_g=3kT_r$ and the value of kT_e obtained from Fig. 4. The detection efficiency $P_d(E_g, E_t)$ is the represented by the shaded area in Fig.5(a):

 $P_{d}(E_{g}, E_{t}) = \int_{E_{t}}^{\infty} F(E) dE.$ (10)



Equation (10) may be evaluated in terms of the tabulated integral, and is plotted (solid curve) in Fig.5(b), again for $E_g=3kT_r$.

To verify that Eqs.(9) and (10) correctly describe our complete detection system, 100 pulses of energy $3 < T_r$ were introduced at random times by the electrostatic force calibrator,² These were detected "blind" by the normal detection algorithm, 98 being found above the (normal) threshold E_{tn} =1.1kT_r. These are plotted (solid points) in Figs.5(a) and 5(b), the agreement being satisfactory.

Programming

Data reduction must start with the 16,384-byte records on magnetic tape, a pair of bytes representing the x and y components of the oscillation amplitude in a frame rotating at the frequency of the reference oscillator. The oscillator. reference the feed detectors phase-sensitive integrators with an infinite relaxation time, which are reset after each sampling Impulse Thus, for Instance, Interval.

noise affects only a single sampling interval.

Since a computer was necessary in any case to reduce data, we designed the experiment to preserve as much of the original information as possible, and the use of integrators without interval-to-interval memory is in accord with this philosphy. The actual data reduction program then has a number of steps:



of a program and another to produce a program which actually performs these tasks and no others. The program was written by the authors in APL, a powerful Interactive system. In order to test the processing programs, mock data were prepared by a completely different set of APL programs. These programs accept the parameters of the experiment it was desired to simulate (F_B, F_0, δ) , Tr, and provide at first a set of etc.) thermal impulses, assign arbitrary phase angles with respect to the reference oscillator and with amplitudes drawn from a Boltzmann distribution appropriate to the experiment. Another program (CALSIM) could be used to generate the linear consequences of applying a calibrating Impulse of arbitrary amplitude at any time. In this way, the processing programs were thoroughly tested, a few their efficiency removed, and bugs they returned the known increased until the simulated gravity parameters of data where no For simulated waves. simulated gravity waves were introduced, the output was satisfactorily Boltzmann distributed.

At this point, a programmer was hired to turn the APL programs into faster-running FORTRAN programs, which were then tested on the same mock-data streams, and a few bugs removed until the outputs from the FORTRAN programs and the APL programs were identical. When real data from the gravity wave antenna became available, the computer programs were used to provide information about the apparatus. The first few minutes of observation showed that the energy autocorrelation function was almost zero from 100 milliseconds to about 150 milliseconds. At this time, the servo isolation support had not been incorporated, and the problem was that the amplitude of oscillation of the transducer due to building vibrations was sufficient to drive the preamplifier into saturation periodically and thus to produce wide zeros in the autocorrelation function. Reduction of the transducer gain below a few hundred hertz, together with the use of the pneumatic support, eliminated this problem.

Days of operation with the data recorder looking at a signal generator rather than at the gravity wave apparatus showed about one interval per day with a substantial amplitude, d(t). This was due to an Interchange between x and y channels, caused by the fact that the reference oscillator for the lock-in-detectors and a reference clock in the data interface were asynchronous. A logical flaw in the design of the interface was uncovered and remedied by redesign. We also provided a programming modification which, although Introduced to detect and automatically reject "popcorn noise" from the input transistor, also would have rejected this data interface pulse had we not been able to remedy the hardware. The program applied automatically a test of reasonableness to the successive vector amplitudes v_n, noting that, whatever the arrival time of an excitation within the sampling interval, the vector \boldsymbol{v}_{p} should not lle very far outside a band connecting v_{p-1} and v_{p+1} .

only the two Finally, although quadrature data channels were recorded, we wanted some protection against people banging on the apparatus, and so we provided a delayed marker in the data themselves, whereby preamplifier output the channels was wlde-band monitored before the lock in detectors, and when a threshold was exceeded, the data channels were blanked in а distinctive pattern beginning two sampling Intervals after the detection.

The computer output included the number of blanks and bursts (popcorn noise) and printed out the details of the neighborhood of these occurrences. We wanted all determinations to be made automatically, so that no data-dependent blas could be introduced by the experimenters. In fact, we never had occasion to question the computer's judgment in rejecting these points.

Callbration

Calibration pulses were introduced about once a day at the wish of the experimenter, and were recovered by the standard computer program. Even large calibration pulses driving the bar to 6000 K of energy did not trigger the burst detector nor blank the data channels by detection ir the wide-band signal. Therefore we do not believe that any gravity waves were rejected in this way. In order to determ sensitivity and efficiency the determine of the no detailed analysis The mass and length of apparatus, 15 required. The mass and length of the bar, the diameter and spacing of the electrostatic calibrating plate, and the amplitude and number of cycles of the square-wave calibrating pulse provide all the data required to determine efficiency for detection of pulse gravitational radiation of any assumed wave form (see Eq.(7)). The experimental histogram of impulses can then be interpreted in terms of the expected Boltzmann distribution, and any excess taken as an upper limit to the rate of incidence of gravitational radiation.

Experience

The data of physical interest have already been published. 2,3,4 We have reviewed the performance of the apparatus for the 27 days of data taken December 1973. Over this period the Q of the apparatus increased uniformly by about 5%. (v²) (bar thermal energy plus amplifier noise within the bandpass of the lock-ins) was 329 ± 4 (rms) K over the 27 days, with no significant trend. The effective noise (kT_e) dropped from 18.6 K to about 17.4 K uniformly over the

month, We have evaluated the performance of the computer determination of $(f_B - f_0)$ which is used automatically to derotate the data to avoid impulses appearing simply to due to frequency offset of the reference oscillater. The rms accuracy in determining the phase advance appears to be about 0.004 radians, contributing about 0.04 K to the 18 K T_e of our

experiment.

We ran our experiment again for about a month in January 1975, without adjustment from the previous run in December 1973. A similar Boltzmann distribution resulted, the parameters of the apparatus having changed by less than 2 percent.

Our data reduction programs have not changed since mid-1973. We make no claim that our system is optimum in any mathematical sense. We do claim that its performance has been simply demonstrated by our dependence on the electrostatic calibrator. As we have shown, an

otherwise excellent experiment⁷ suffered about a factor 10 unnecessary loss in sensitivity by the employment of a 0.3 sec filter on the amplitude components, before sampling at 0.1 sec interval.⁸ In addition, these workers had an excess of impulses in the tail of the Boltzmann distribution, because they did not derotate the data by computer and did not have sufficiently precise tracking of the bar frequency with the reference oscillator. Thus for gravity waves incident at the rate of one per day their sensitivity was about 2 kT_r, whereas the elimination of the 0.3-sec filters would have provided a sensitivity of about 0.19 kT_r.

Conclusion

We designed our experiment so that different elements of it (antenna, data Interface, transducer, computer processing, simulation packages) could be tested against one another. We confident that we could not design We were and build an experiment without error the first time. Therefore, we concentrated on testing and Improving--wherever possible under circumstances where we gravitational were not sensitive to radiation. To a large extent, our confidence in the results is due to the simple nature of the apparatus made possible by use of computer processing, by the dependence upon simple and transparent simulation program, and by the ultimate reliance on the absolute nature of electrostatic calibration to determine sensitivity and noise performance.

1 J. Weber, Nature (London) 240, 28 (1972).

²J.L. Levine and R.L. Garwin, Phys.Rev.Lett. <u>31</u>, 173 (1973).

³*R.L.* Garwin and J.L. Levine, *Phys.Rev.Lett.* <u>31</u>, 176 (1973).

⁴J.L. Levine and R.L. Garwin, Phys.Rev.Lett. <u>33</u>, 794 (1974).

⁵R.L. Garwin, Physics Today, 9 (December 1974).

⁶Refs. 2 and 3 report results with a 120-kg bar. Some of the numerical data given here are for the 120-kg bar.

⁷D. Bramanti, et al, Lett. [.] Nuovo Cimento <u>7</u>, 665 (1973).

⁸J.L. Levine, Lett. Nuovo Cimento <u>11</u>, 280 (1974).

DISCUSSION

Maeder: I have two questions. One is the Q-factor of the detector: to which extent is this affected by the different devices mounted at the ends?

Garwin: Well, the initial Q of the detector was about 300,000 without any transducer on it. The loaded transducers have an inherent Q of the material (Q_M) of about 1000 and so, if one transfers the energy into electric energy, naturally the Q of the detector is reduced. Our first detector of 120 kg had a loaded Q of 4200 which was negatively affected by the electrostatic calibrator on the other end and our 500 kg detector had a loaded Q up to about 40,000.

Maeder : Is it possible to express the efficiency of the transducer by a factor which compares to the β value that is used by other workers? What counts for the overall sensitivity is the product $\beta Q_{...}$

Garwin : Yes, that is right. Our β is 2%. Is it a reasonable number?

Maeder : 2% is a reasonable number for a β value if you have a low Q.

Garwin: We have $\beta Q \leq \epsilon Q_M$ where β is the fraction of bar oscillation energy present as electrical energy in the transducer and ϵ is the electro-mechanical coupling of the transducer material (0,4 - 0,7). Our βQ was 100 - 200, But we do not use these numbers very much; what we use is the calibrating pulse, in order to determine the energy scale and I can go over this with you privately.

Maeder : Then I have a final suggestion. You mentioned an effective noise temperature of 18°K (after the filtering) and from this I conclude that you are close to optimum filtering assuming that the noise temperature of your amplifier is about 1°K, because one can show that the effective noise temperature of the overall system is the geometric mean of the detector temperature which is 300°K and the amplifier noise temperature ture

$$T_{\text{opt}} = \sqrt{T_{\text{detector}}} \cdot T_{\text{amplifier}} \cong \sqrt{300^{\circ}K \cdot l^{\circ}K} = \sim 18^{\circ}K$$

Do you agree with this interpretation?

Garwin: Yes, that is quite right. What one wants to do is to choose the sampling time τ so as to minimize the overall noise. The noise comes in two ways, one at 300°K from the bar, and the other contribution from the wide band amplifier. The choice of sampling time is indeed determined by such considerations, long τ leading to a greater influence of 300°K bar oscillation noise and short τ a preponderance of wide-band amplifier noise [see eqs. (5) and (6)]. If you change any of these parameters you get only the square root because you reoptimize as you said.